ГОДИШНИК НА СОФИЙСКИЯ УНИВЕРСИТЕТ „СВ. КЛИМЕНТ ОХРИДСКИ“

ФАКУЛТЕТ ПО МАТЕМАТИКА И ИНФОРМАТИКА
Том 106

ANNUAL OF SOFIA UNIVERSITY „ST. KLIMENT OHRIDSKI“

FACULTY OF MATHEMATICS AND INFORMATICS
Volume 106

NONEXISTENCE OF (17, 108, 3) TERNARY ORTHOGONAL ARRAY

SILVIA BOUMOVA, TANYA MARINOVA, TEDIS RAMAJ, MAYA STOYANOVA

We develop a combinatorial method for computing and reducing of the possibilities of distance distributions of ternary orthogonal array (TOA) of given parameters (n, M, τ). Using relations between distance distributions of arrays under consideration and their relatives we prove certain constraints on the distance distributions of TOAs. This allows us to collect rules for removing distance distributions as infeasible. The main result is nonexistence of $(17, 108, 34)$ TOA. Our approach allows substantial reduction of the number of feasible distance distributions for known arrays. This could be helpful for other investigations over the classification of the ternary orthogonal arrays.

Keywords: Hamming space, orthogonal arrays, Krawtchouk polynomials, distance distributions.

2010 Math. Subject Classification: Primary: 05B15; Secondary: 94B25.

1. INTRODUCTION

Let $H(n, 3)$ be the Hamming space of dimension n over the alphabet $\{0, 1, 2\}$. The Hamming distance $d(x, y)$ between two points $x, y \in H(n, 3)$ is equal to the number of coordinates where they differ.

Definition 1.1. An orthogonal array (OA) of strength τ and index λ in $H(n, 3)$ (also called ternary orthogonal array or TOA), consists of the rows of an $M \times n$ matrix C with the property that every $M \times \tau$ submatrix of C contains all ordered τ-tuples of $H(\tau, 3)$, each one exactly $\lambda = M/3^\tau$ times as rows. We denote such orthogonal array as (n, M, τ) TOA.

Let $C \subset H(n,3)$ be an (n,M,τ) TOA and $c \in H(n,3)$ is a fixed point of the space.

Definition 1.2. The distance distribution of C with respect to the point c is the $(n+1)$-tuple $W = W(c) = (w_0, w_1, \dots, w_n)$, where

$$w_i = |\{x \in C \mid d(x,c) = i\}|, \quad i = 0, \dots, n.$$

If $w_0 \geq 1$ then the point c is a word in the array C and such points we denote as internal points. The case $w_0 = 0$ denote an external point for the orthogonal array C. For simplicity and differentiation the distance distributions of internal and external points will be denoted as $P = P(c) = (p_0, p_1, \dots, p_n)$ and $Q = Q(c) = (q_0, q_1, \dots, q_n)$, respectively.

Let n, M and $\tau \leq n$ be fixed. The sets of all possibilities for distance distributions of a given (n,M,τ) TOA with respect to internal points and external points are denoted by $P(n,M,\tau)$ and $Q(n,M,\tau)$, respectively. Their union is the set $W(n,M,\tau) = P(n,M,\tau) \cup Q(n,M,\tau)$.

There is a method [6, 2] for computation of the sets $P(n,M,\tau)$, $Q(n,M,\tau)$ and $W(n,M,\tau)$. This method is based on the fact that each orthogonal array is a design in $H(n,3)$.

We consider the Hamming space $H(n,3)$ as polynomial metric space where zonal orthogonal polynomials are the Krawtchouk polynomials. For fixed n and $q = 3$, the (normalized) Krawtchouk polynomials are defined by

$$Q_i^{(n)}(t) := \frac{1}{r_i} K_i^{(n,3)}(z),$$

where $z = n(1-t)/2$, $r_i := 2^i \binom{n}{i}$, and

$$K_i^{(n,3)}(z) := \sum_{j=0}^{i} (-1)^j 2^{i-j} \binom{z}{j} \binom{n-z}{i-j},$$

$i = 0, 1, \dots, n$, are the (usual) Krawtchouk polynomials [1, 14].

Definition 1.3. [10] A code $C \subset H(n,3)$ is a τ-design if and only if for every real polynomial $f(t)$ of degree at most τ and for every point $c \in H(n,3)$ the equality

$$\sum_{x \in C} f(\langle c, x \rangle) = f_0 |C|$$

holds, where f_0 is the first coefficient in the expansion $f(t) = \sum_{i=0}^{n} f_i Q_i^{(n)}(t)$ and $\langle c, x \rangle = 1 - 2d(c,x)/n$.

Since every (n,M,τ) TOA is a τ-design, the following theorem holds.

Theorem 1.4 ([6, 2]). *Let $C \subset H(n,3)$ is an (n,M,τ) TOA and $c \in H(n,q)$ is a fixed point. The following propositions are valid*

(a) If $c \in C$, for the distance distribution of C with respect of c the following system holds:

$$\sum_{i=0}^{n} p_i \left(1 - \frac{2i}{n}\right)^k = b_k|C|, k = 0, 1, \ldots, \tau, \tag{1.1}$$

(b) If $c \notin C$, for the distance distribution of C with respect of c the following system holds:

$$\sum_{i=1}^{n} q_i \left(1 - \frac{2i}{n}\right)^k = b_k|C|, k = 0, 1, \ldots, \tau, \tag{1.2}$$

where $b_k = f_0$ is the first coefficient in the expansion of the polynomial t^k by the normalized Krawchouk polynomials.

Through this theorem all initially feasible distance distributions of TOA of parameters (n, M, τ) can be computed effectively for relatively small n and τ.

Boyvalenkov and two of authors [4] have presented and implemented an algorithm for investigation binary orthogonal arrays. In this paper we develop a similar algorithm that reduces the possible elements of the set $P(n, M, \tau)$. This algorithm uses some connections between a given TOA and its related TOAs. During the implementation of the algorithm this set $P(n, M, \tau)$ is changed by ruling out some distance distributions.

In Section 2 we prove several relations between distance distributions of arrays under consideration and their relatives. This imposes significant constraints on the targeted TOAs and allows us to collect rules for removing distance distributions from the set $P(n, M, \tau)$. The algorithm and one nonexistence result are described in Section 3.

2. RELATIONS BETWEEN DISTANCE DISTRIBUTIONS OF (N, M, τ) TOA AND ITS DERIVED

Let n, M and $2 \leq \tau < n$ be fixed. Let $C \subset H(n, 3)$ be an (n, M, τ) TOA with sets of distance distributions $P(n, M, \tau)$, $Q(n, M, \tau)$ and $W(n, M, \tau)$ after calculating the results of the systems (1.1) and (1.2). We proceed with the removing a column from C. Using well-known properties of the orthogonal arrays [8] we obtain another orthogonal array C' with the same strength and cardinality and length $n-1$. Without loss of generality (see [8]) let $P \in P(n, M, \tau)$ be the distance distribution of C with respect to $c = \mathbf{0} \in C$. Then the point $c' = \mathbf{0} \in C'$ and the distance distribution of C' with respect to c' we denote by $P' \in P(n-1, M, \tau)$. The scheme of this construction is shown in the Figure 1 bellow.

Definition 2.1. For every $i \in \{0, 1, \dots, n\}$ the submatrix which consists of the rows of C with i nonzero coordinates is called an i-block.

It follows from the definition of distance distribution that the i-block is a $w_i \times n$ matrix. Next we denote by y_i the number of the zeros in the intersection of the fixed column of C and the rows of the i-block. The number of the nonzero elements in this intersection is denoted by $\overline{y}_i$.

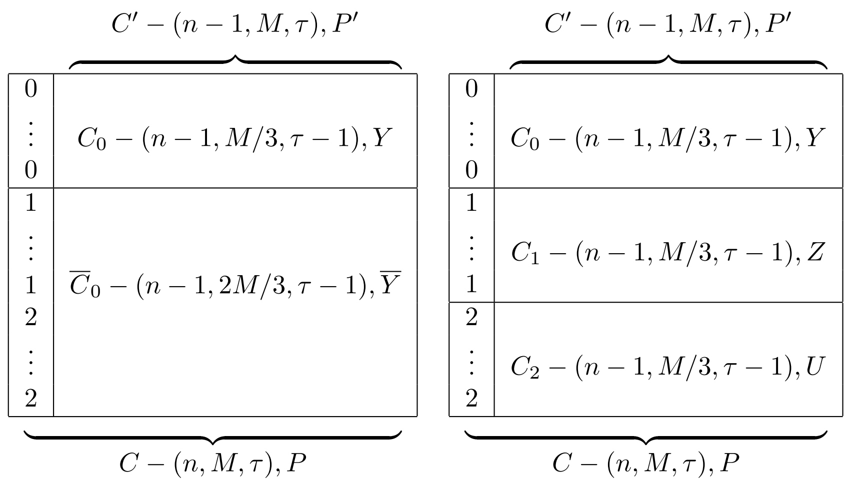

Figure 1

Theorem 2.2. *The nonnegative integer numbers y_i and $\overline{y}_i$, for $i = 0, 1, \dots, n$, satisfy the following system of linear equations*

$$\left| \begin{array}{l} y_i + \overline{y}_i = p_i, \ \ i = 1, 2, \dots, n-1 \\ y_i + \overline{y}_{i+1} = p'_i, \ \ i = 0, 1, \dots, n-1 \\ y_0 = p_0, \ \overline{y}_n = p_n \\ \overline{y}_i, y_i \in \mathbb{Z}, \ \ x_i \geq 0, \ \ y_i \geq 0, \ \ i = 0, 1, \dots, n \end{array} \right. . \tag{2.1}$$

Proof. From the definition of the numbers y_i and $\overline{y}_i$ directly we obtain the equalities:

$$y_i + \overline{y}_i = p_i, \quad i = 1, \dots, n-1, \quad \overline{y}_n = p_n, \quad y_0 = p_0.$$

Let us have a look at the i-th element p'_i in the distance distribution P' of C' with respect to $c' = \mathbf{0} \in C'$. It denotes the number of points in C' that have exactly i nonzero coordinates. Such points can be obtained from C by removing the first column in two possible ways. The first one is from a point which first coordinate is zero and has i nonzero entries. The number of these words of C is exactly y_i. Second is from a point of C with $i+1$ nonzero entries such that one of them is in the first column. These are the points in the $(i+1)$-block and their number is $\overline{y}_{i+1}$. Therefore

$$y_i + \overline{y}_{i+1} = p'_i$$

for every $i = 0, 1, \ldots, n-1$. $\square$

Remark 2.3. There is a generalization of Theorem 2.2, i.e. the assertion is valid not only for internal points but also for every distance distribution $W \in W(n, M, \tau)$. However, for the purposes of the algorithm in the next section we can limit to the distance distributions in $P(n, M, \tau)$.

Corollary 2.4. *The distance distribution $P \in P(n, M, \tau)$ is not feasible if no system (2.1) obtained when P' runs $P(n-1, M, \tau)$ has a solution.*

Corollary 2.4 rules out some distance distributions P but its main purpose is to define feasible pairs (P, P') which we will investigate further.

If we order the elements in some column (for example the first column) of (n, M, τ) TOA C and remove this column (as shown in the Figure 1) we obtain three different $(n-1, M/3, \tau-1)$ TOAs. One of them is C_0 – the TOA obtained from C by ordering the zeros in the first column and taking the corresponding points of C'.

Theorem 2.5. *The vector Y is the distance distribution of C_0 with respect to the internal point $c' = \mathbf{0} \in C'$, i.e. $Y = (y_1, y_2, \ldots, y_{n-1}) \in P(n-1, M/3, \tau-1)$.*

Proof. We know from the definition of i-block that y_i is exactly the numbers of points in C with distance i to the fixed point $c = \mathbf{0} \in C$. Therefore, the number of points in C_0 with distance i to the point $c' = \mathbf{0} \in C'$ is exactly y_i. $\square$

Corollary 2.6. *If $Y \notin P(n-1, M/3, \tau-1)$ then the pair (P, P') is not feasible.*

Let us return to the construction in Figure 1. We denote by C_1 and C_2 the orthogonal arrays corresponding to the sorted and removed elements one and two in the first column of C, respectively. Another property of the orthogonal arrays says that an union of C_1 and C_2, two ternary orthogonal arrays with parameters $(n-1, M/3, \tau-1)$ and $(n-1, M/3, \tau-1)$ is also a TOA with parameters $(n-1, 2M/3, \tau-1)$. This union will be denoted by $\overline{C}_0$. Note that there may be repeating points in the considered orthogonal arrays.

Theorem 2.7. *If $\overline{y}_0 \geq 1$, then $\overline{Y}$ is the distance distribution of $\overline{C}_0$ with respect to the fixed point $c' = \mathbf{0}$, i.e. $\overline{Y} \in P(n-1, 2M/3, \tau-1)$.*

Proof. The nonzero entries of the first column of C are selected and removed and this way the orthogonal array $\overline{C}_0$ is obtained. We have from the definition of i-block that $\overline{y_i}$ is exactly the numbers of points in C with distance i to the point $c = \mathbf{0}$. Therefore, the numbers of points in $\overline{C}_0$ with distance i to the point $c' = \mathbf{0}$ is exactly $\overline{y}_i$. The condition $\overline{y}_0 \geq 1$ determines that we check if $\overline{C}_0$ contains the point $c' = \mathbf{0}$, i.e. $c' \in \overline{C}_0$ is the internal point and $\overline{Y} \in P(n-1, 2M/3, \tau-1)$. $\square$

Corollary 2.8. *If* $\overline{y}_0 \geq 1$ *and* $\overline{Y} \notin P(n-1, 2M/3, \tau-1)$, *then the pair* (P, P') *is not feasible.*

After applying Corollary 2.8 for fixed distance distribution $P \in P(n, M, \tau)$ we continue with the remaining feasible pairs (P, P'). Let

$$(\overline{y}_0^{(r)} = 0, \overline{y}_1^{(r)}, \ldots, \overline{y}_n^{(r)}; y_0^{(r)}, y_1^{(r)}, \ldots, y_{n-1}^{(r)}, y_n^{(r)} = 0), \quad r = 1, \ldots, s,$$

be all solutions of system (2.1) when P' runs the set $P(n-1, M, \tau)$ such that the corresponding pair (P, P') is not ruled out by Corollaries 2.6 and 2.8. Denote by k_r the numbers of columns corresponding to the r-th solution of the system (2.1) for $r = 1, \ldots, s$.

After the sieve from Corollaries 2.6 and 2.8, we formulate another necessary condition for the existence of C.

Theorem 2.9. *The system*

$$\left| \begin{array}{lllll} k_1 & +k_2 & +\ldots & +k_s & = n \\ k_1\overline{y}_1^{(1)} & +k_2\overline{y}_1^{(2)} & +\ldots & +k_s\overline{y}_1^{(s)} & = p_1 \\ k_1\overline{y}_2^{(1)} & +k_2\overline{y}_2^{(2)} & +\ldots & +k_s\overline{y}_2^{(s)} & = 2p_2 \\ \vdots & & & & \\ k_1\overline{y}_n^{(1)} & +k_2\overline{y}_n^{(2)} & +\ldots & +k_s\overline{y}_n^{(s)} & = np_n \\ k_j \in \mathbb{Z}, & k_j \geq 0, & j = 1, & \ldots, s & \end{array} \right. \tag{2.2}$$

with respect to $k_1, k_2, \ldots, k_s$ *has a solution, i.e. the ternary orthogonal array* C *of parameters* (n, M, τ) *exists if the system (2.2) has a solution.*

Proof. For every cutting of a column of C we solve the system (2.1) for every possible $P' \in P(n-1, M, \tau)$. Let i be fixed. In the i-block the numbers of nonzero entries is exactly ip_i. On the other hand we know that $\overline{y}_i$ is the number of points in i-block with entries 1 or 2 in the first column. So the count of nonzero entries in i-block is equal to $k_1\overline{y}_i^{(1)} + k_2\overline{y}_i^{(2)} + \ldots + k_s\overline{y}_i^{(s)}$. Therefore for every $i = 0, \ldots, n$ the equalities in the system (2.2) hold. $\square$

3. OUR ALGORITHM AND ONE NONEXISTENCE RESULT

Based on the observations and conclusions in the previous section an algorithm for reducing the feasible distance distributions in the set $P(n, M, \tau)$ for fixed n, M and τ can be developed. If the result from the algorithm is an empty set we can conclude that ternary orthogonal arrays with parameters (n, M, τ) do not exist.

By calculating the sets $P(n-1, 2M/3, \tau-1)$ we observe that these sets become very large so the Theorem 2.7 and the Corollary 2.8 are not easy to be applied for the computations. Even more, when $\tau > 3$ the set $P(n-1, 2M/3, \tau-1)$ itself should be reduced through generation and reduction of the set $P(n-2, 4M/9, \tau-2)$ which cardinality is even bigger. That is the reason why the algorithm is based only on Theorems 2.2, 2.5 and 2.9 and their corollaries.

First, we generate with Theorem 1.4 the following rows of distance distribution sets when the length varies from τ to n

$$P(\tau, M, \tau), P(\tau+1, M, \tau), \dots, P(n, M, \tau)$$

$$P(\tau-1, M/3, \tau-1), P(\tau, M/3, \tau-1), \dots, P(n-1, M/3, \tau-1)$$

$$\dots$$

For fixed j, $j = \tau, \tau+1, \dots, n$, the algorithm is applied over the set $P(j, M, \tau)$ and its derived as the algorithm ends either if $j = n$ or if an empty set is obtained for some j.

From the set $P(j, M, \tau)$ a distance distribution P is selected. For this fixed distance distribution and for every distance distribution in $P' \in P(j-1, M, \tau)$ the system (2.1) is resolved. If for every P' this system does not have a solution, the distance distribution P is ruled out from $P(j, M, \tau, 3)$, (see Corollary 2.4).

Otherwise, for the solution $(Y, \overline{Y})$ we check the condition in Theorem 2.5. If it is not fulfilled the pair (P, P') is not feasible (see Corollary 2.6).

For every feasible (P, P') we collect the solution $(Y, \overline{Y})$. After all solutions are collected when P' runs over $P(j-1, M, \tau)$ the system (2.2) is solved. If there is no solution, the distance distribution P is ruled out from the set $P(j, M, \tau)$, (see Theorem 2.9).

This is the step for fixed j. After reducing the elements of $P(j, M, \tau)$ we increase j by 1 and proceed with the next set of investigation of the distance distributions $P(j+1, M, \tau)$. We continue until $j < n$. If the set $P(j, M, \tau)$ is empty for some $j_0 < n$ the algorithm ends with the conclusion that (j, M, τ) TOAs do not exist for $j = j_0, j_0+1, \dots, n$.

For the sake of clarity a pseudocode of the algorithm is provided bellow.

In what follows, our investigation is focused on the set $P(17, 108, 3)$, one of the cases in [11] where the existence was marked as undecided. Moreover, for $n = 12, \dots, 17$ there are no evidence whether orthogonal arrays with parameters $(n, 108, 3)$ exist. Several teams of authors ([5, 12, 13]) have investigated these among many others cases, but the issue of the existence of a ternary orthogonal array with parameters $(17, 108, 3)$ has not been clarified so far.

We calculate all possible distance distributions for internal point, i.e. we generate the sets $P(n, 108, 3)$ for $n = 3, \dots, 17$. Along with this we need the sets $P(n, 36, 2)$ for $n = 2, \dots, 16$. The Algorithm 1 is applied on these sets. First the sets $P(j, 36, 2)$ are reduced, starting from $j = 2$. After that the sets $P(n, 108, 3)$ are reduced. Then last reduced set is $P(17, 108, 3)$. In the tables below the cardinalities of all these sets are provided. In the first table the results for $|P(n, 108, 3)|$ are presented for $n = 3, 4, \dots, 17$. The entry $A \to B$ in the first table means that

Algorithm 1 Algorithm over TOAs

```
procedure NDDA(P(n, M, τ), P(n − 1, M, τ), P(n − 1, M/3, τ − 1))
    Input: n, M, τ, P(n, M, τ), P(n − 1, M, τ), P(n − 1, M/3, τ − 1)
    filteredP ← empty set
    for P ∈ P(n, M, τ) do
        allȲ ← empty set
        for P′ ∈ P(n − 1, M, τ) do
            Y, Ȳ ← solve system (2.1) for integer nonnegative solutions
            if no integer solutions then
                next;
            if Y ∈ P(n − 1, M/3, τ − 1) then
                add Ȳ to allȲ
        if allȲ is empty then
            add P to filteredP
        else
            if system (2.2) has no integer nonnegative solutions then
                add P to filteredP
    Output: P(n, M, τ) \ filteredP
```

in the beginning there is A initially feasible distance distributions of $(n, 108, 3)$ TOA, i.e. the set $P(n, 108, 3)$ has A elements, starting from $n = 3$ in the first row and the first column and ending to $n = 17$ in the third row and the fifth column, successively. The number B after the arrow (in corresponding entry) represents the reduced value B of elements in the set $P(n, 108, 3)$ in the end of the algorithm, $n = 3, 4, \dots, 17$. Analogously, the results for $|P(n, 36, 2)|$ are presented in the second table for $n = 2, 3, \dots, 16$ and $\tau = 2$.

$\lvert P(n, 108, 3)\rvert$:	$1 \to 1$	$4 \to 4$	$18 \to 16$	$48 \to 43$	$113 \to 89$
	$271 \to 208$	$440 \to 368$	$701 \to 540$	$1002 \to 702$	$879 \to 699$
	$901 \to 660$	$631 \to 337$	$119 \to 29$	$49 \to 6$	$10 \to 0$

$\lvert P(n, 36, 2)\rvert$:	$1 \to 1$	$4 \to 4$	$16 \to 14$	$31 \to 30$	$52 \to 49$
	$85 \to 79$	$109 \to 105$	$121 \to 109$	$127 \to 111$	$108 \to 100$
	$85 \to 79$	$62 \to 50$	$28 \to 26$	$12 \to 11$	$6 \to 4$

The zero in the last element $10 \to 0$ of the first table corresponds to the the number of elements in the set $P(17, 108, 3)$, i.e. our algorithm ends with the empty set $P(17, 108, 3) = \emptyset$. Therefore, we obtain the following nonexistence result.

Theorem 3.1. *There exist no ternary orthogonal array of parameters* $(17, 108, 3)$.

All calculations in this paper were performed by programs in Maple. All results (in particular all possible distance distributions in the beginning) can be seen at [15]. All programs are available upon request.

ACKNOWLEDGEMENTS. The research of the first author was supported, in part, by the Bulgarian National Science Fund (NSF) under contract KP-06-N32/1-2019. The research of the third author was supported, in part, by the Bulgarian NSF under contract DN2-02/2016. The research of the fourth author was supported, in part, by the Bulgarian NSF under contract KP-06-N32/2-2019.

4. REFERENCES

[1] Abramowitz, M., Stegun, I. A.: *Handbook of Mathematical Functions, with Formulas, Graphs, and Mathematical Tables*. New York, Dover, 1964.

[2] Boyvalenkov, P., Kulina, H.: Investigation of binary orthogonal arrays via their distance distributions. *Probl. Inf. Transm.*, **49(4)**, 2013, 320–330.

[3] Boyvalenkov, P., Kulina, H., Marinova, T., Stoyanova, M.: Nonexistence of binary orthogonal arrays via their distance distributions. *Probl. Inf. Transm.*, **51(4)**, 2015, 326-334.

[4] Boyvalenkov, P., Marinova, T., Stoyanova, M.: Nonexistence of a few binary orthogonal arrays. *Discrete Appl. Math.*, **217(2)**, 2017, 144–150.

[5] D. A. Bulutoglu, D. A., Margot, F.: Classification of orthogonal arrays by integer programming. *J. Statist. Plann. Inference*, **138**, 2008, 654–666.

[6] Delsarte, P.: An Algebraic Approach to the Association Schemes in Coding Theory. *Philips Res. Rep. Suppl.*, **10**, 1973.

[7] Delsarte, P., Levenshtein, V. I.: Association schemes and coding theory. *IEEE Trans. Inform. Theory*, **44**, 1998, 2477–2504.

[8] Hedayat, A., Sloane, N. J. A., Stufken, J.: *Orthogonal Arrays: Theory and Applications*. Springer-Verlag, New York, 1999.

[9] Levenshtein, V. I.: Krawtchouk polynomials and universal bounds for codes and designs in Hamming spaces. *IEEE Trans. Infor. Theory*, **41**, 1995, 1303–1321.

[10] Levenshtein, V. I.: Universal bounds for codes and designs. In: *Handbook of Coding Theory*. (V. S. Pless and W. C. Huffman, Eds.), **Ch. 6**, Elsevier, Amsterdam, 1998, 499–648.

[11] http://neilsloane.com/oadir/index.html

[12] Schoen, E.D., Eendebak, P. T., Nguyen, M. V. M.: Complete enumeration of pure-level and mixed-level orthogonal arrays. *J. Combin. Des.*, **18**, 2009, 123–140.

[13] Seiden, E., Zemach, R.: On orthogonal arrays. *Ann. Math. Statist.*, **37**, 1996, 1355–1370.

[14] Szegő, G.: *Orthogonal Polynomials*. American Mathematical Society Colloquium Publications 23, AMS, Providence, RI, 1939.

[15] https://store.fmi.uni-sofia.bg/fmi/algebra/stoyanova/toa.html

Received on February 6, 2020
Received in a revised form on March 17, 2020

Silvia Boumova

Faculty of Mathematics and Informatics
"St. Kliment Ohridski" University of Sofia
5 blvd. J. Bourchier, BG-1164 Sofia
BULGARIA

E-mail: boumova@fmi.uni-sofia.bg

Tanya Marinova

Faculty of Mathematics and Informatics
"St. Kliment Ohridski" University of Sofia
5 blvd. J. Bourchier, BG-1164 Sofia
BULGARIA

E-mail: tanya.marinova@fmi.uni-sofia.bg

Tedis Ramaj

Faculty of Mathematics and Informatics
"St. Kliment Ohridski" University of Sofia
5 blvd. J. Bourchier, BG-1164 Sofia
BULGARIA

E-mail: tramaj@fmi.uni-sofia.bg

Maya Stoyanova

Faculty of Mathematics and Informatics
"St. Kliment Ohridski" University of Sofia
5 blvd. J. Bourchier, BG-1164 Sofia
BULGARIA

E-mail: stoyanova@fmi.uni-sofia.bg