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LIFE AND WORKS OF DIMITER SKORDEV

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We sketch the life of Dimiter Skordev and his main mathematical works. He invented the recursively complete functions and the combinatory spaces. He obtained important results in subrecursive analysis and many other mathematical areas.

Keywords: Dimiter Skordev, mathematical logic, computability, subrecursive analysis, history of Bulgarian mathematics

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1. Prologue

People talking vanity squander their words and belittle themselves. People talking about substantial and perpetual matters participate in the glorious things they talk about. We participate in the glory of the deceased man when we praise his deeds. And through our words of praise his glory perpetuates.

On December 29, 2022 our teacher Dimiter Skordev passed away. Ours is the job to write about what we are unwilling to accept. The job is hard but also good and worthy, it honors us and we proceed.

Dimiter Skordev was born on June 1, 1936 in an illustrious family. His parents, renowned architects, bestowed gratuitously the projects of several meteorological stations and huts, situated on the heights of the Bulgarian mountains. And so would do their child, Dimiter, who bestowed gratuitously his life to his students, bringing them along to the heights of the mathematical science.

Skordev finished secondary school ahead of time. In 1953 at the age of 17 he became a student in the Faculty of Physics and Mathematics of Sofia University. Several mathematical courses at Sofia University were taught in a modern, logically clear way. They raised in Skordev the interest towards logical problems and

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prepared a ground for further acquaintance with their contemporary treatment. In his Historical Notes [117, p. 12] Skordev mentioned specifically two such courses: "Analysis" taught by Prof. Tagamlitzki, who introduced the real numbers not by means of a definition but axiomatically, and "Foundations of Mathematics" taught by Prof. Petkanchin, which was especially helpful, thanks to its subject matter and the irreproachable way of its presentation.

At that time the scientists in Bulgaria were largely isolated from the rest of the world. The renowned seminar of Prof. Tagamlitzki became a center of intensive and fruitful scientific activity and the only place where students and young mathematicians could acquaint themselves with the contemporary state of the science, to work on unsolved problems, in short – to make their first steps as young scientists. Some of the results achieved in this seminar, even by students, were reported at mathematical conferences and published in scientific magazines.

Skordev was awarded several state prizes for achievements in this seminar; he won his first prize while being only a second year student. In the written opinion of Prof. Petkanchin about one of the works of Skordev it is said that Skordev was universally "recognized as a student with broad mathematical interests and knowledge, who has shown tenacity and thoroughness in the study of mathematical problems and has made independent contributions as a member of the seminar."[1]

Skordev got the best from his teachers. Later, as a chief of the Department of Mathematical Logic he would found a logical seminar functioning similarly to the seminar of Prof. Tagamlitzki. It was a place where young logicians and interested students could share their scientific aspirations and achievements in a spirit of cooperation, advancing both themselves and the world of science.

In 1958 Skordev graduated from the Faculty of Physics and Mathematics with distinction. Immediately he became an assistant professor in the Department of Differential and Integral Calculus, whose chief was Prof. Tagamlitzki. By 1960 he had written and submitted the results [83, 84] which he would use for his doctoral thesis.[2]

Skordev was known as the favorite of Prof. Tagamlitzki and he always had affection for his beloved teacher. The impressive number of 18 publications about Prof. Tagamlitzki are written by Skordev.

2. First logical results

The first years after the World War II were not favorable for doing mathematical logic in Bulgaria. The only officially accepted logic was the so called dialectical logic, considered as a part of Marxist-Leninist philosophy. Around 1958–1959 the authorities realized the importance of electronic computing machinery and its relation to mathematical logic (see [149]). Skordev was sent for a one-year specialization

[1] On April 22, 1957.

[2] His doctoral thesis was written only three years later, in 1963, and defended in 1967. We do not know the reasons for this delay.

(1960–1961) at the Department of Mathematical Logic of the Moscow State University. In Moscow he was introduced to various advanced fields of mathematical logic in general and computability theory in particular.

At the same time, A. A. Markov, at his seminar, proposed a problem about the solvability of word equations. Historically, at the end of the 1950's, Markov was the first to ask whether it is decidable if a word equation has a solution. He hoped to obtain a proof for the unsolvability of Hilbert's tenth problem by showing that solvability of word equations is an undecidable problem. The problem given by Markov was solved by Skordev in collaboration with Blagovest Sendov. Markov was impressed and helped the publication of this result in a German magazine [69].

At the seminar of algorithmic set theory Uspensky raised the question of the existence of partial recursive operators that are not μ-recursive. Examples were found by Kuznetsov and Skordev. Shortly after, Skordev published a characterization when a partial recursive operator is also μ-recursive [70]. This was the first non-trivial Bulgarian result in computability theory.

3. Recursively complete functions

In 1961 Skordev returned to Sofia University. Here he began giving some lecture courses in mathematical logic. His course Theory of Recursive Functions was the first lecture course in computability theory in Bulgaria. During the preparation of this course Skordev invented the interesting notion *recursively complete function* [1–6].

Definition 3.1 (1963 [2])**.** A partial function $f\colon \mathbb{N}^2 \to \mathbb{N}$ is *recursively complete* if it is arithmetic and there are constants a, b, c and d, such that for any n, m, k:

- $f(a, n) = n + 1$,
- $f(b, n + 1) = n$,
- $f(f(f(c, n), m), k) = \begin{cases} m, & \text{if } n = 0, \\ k, & \text{if } n \neq 0, \end{cases}$
- $f(f(d, n), m)$ is always defined, and
- $f(f(f(d, n), m), k) = f(f(n, k), f(m, k))$.

Recall that a partial function $f\colon \mathbb{N}^2 \to \mathbb{N}$ is *universal* if it is partial recursive and for any partial recursive function g there exists n, such that $g(m) \cong f(n, m)$ for any m.

Skordev proved that:

- any universal function is recursively complete;
- any recursively complete function encodes all partial recursive functions;
- therefore, any partial function which is partial recursive and recursively complete is a universal function.

With the help of this notion Skordev obtained several very simple definitions of universal functions. Two examples follow:

Example 3.2 (1963 [2]). The following recursively defined partial function f is universal:

- $f(0,0) = 0$,
- $f(0,n+1) = 1$,
- $f(1,n) = n+1$,
- $f(2^{n+1},m) = n$,
- $f(3^{n+1},m) = 2^n 3^m$,
- $f(2^{n+1}3^{m+1},k) = f(f(n,k),f(m,k))$.

Example 3.3 (1970 [4]). Let $p(n,m) = (n+m)(n+m+1)+n-m$. This polynomial has the property that for different n and m the value of p is different. Therefore, we can define recursively the following partial function:

- $f(p(m,0),k) = p(m,k)$,
- $f(p(0,n+1),k) = n$,
- $f(p(m+1,n+1),k) = f(f(n,k),f(m,k))$.

This partial function is universal.

When a recursively complete function is not partial recursive, then the class of functions encoded by it is wider than the class of all partial recursive functions. A natural question arises: to characterize the classes of partial functions that can be encoded by a recursively complete function. In 1965 [3] Skordev proved that a class of partial functions is encoded by a recursively complete function if and only if the class consists of all functions Turing reducible to an oracle.

4. Algebraization of computability theory

4.1. Interim

In 1965, at the age of 29 Skordev became the youngest associate professor in the Faculty. He was still a member of the Department of Differential and Integral Calculus and most of his publications were not in the area of mathematical logic.[3] Nevertheless, in his inaugural lecture “On some algebraic aspects of mathematical logic” [71] he chose to speak about algebraic logic. Interestingly, at that time there were two places where a Bulgarian could study mathematical logic – Moscow and

[3]The authors apologize for not reviewing Skordev’s mathematical results which are not in the area of mathematical logic. We are simply not competent to do so.

Poland. Computability theory was the main area of interest in Moscow and algebraic logic was the area of interest of the Polish school in mathematical logic. Despite Skordev's specialization in Moscow, his inaugural lecture shows that he had followed closely the achievements of the Polish school. In fact, bringing algebraic methods to computability theory would become Skordev's monumental mathematical achievement.

In 1970 Sofia University and the Bulgarian Academy of Sciences formed United Center of Mathematics and Mechanics. A Sector of Topology and Mathematical Logic was created. Skordev became its member.

In 1972 the chair of the Department of Mathematical Logic of the Moscow State University, A. A. Markov, visited Bulgaria. He delivered several lectures in recursive mathematical analysis. Markov was infuriated by the fact that there was no separate sector of mathematical logic. On the very same day he left Bulgaria, a separate Sector of Mathematical Logic was created and Skordev became its chief. Initially there were five members.

4.2. The discovery of the combinatory spaces

In 1969 Yiannis Moschovakis [146] invented an abstract notion of computability in abstract structures. Unlike the common computability over naturals, this generalization required multi-valued functions to be used instead of single-valued partial functions.

Skordev was intrigued by the fact that far too often in computability theory one can work with multi-valued functions more or less in the same way as with partial single-valued functions. He carried a systematic research of phenomena related to the multi-valued functions.

Skordev invented a new generalization of the computable functions, which was equivalent to the generalization of Moschovakis. Skordev's generalization was, however, much simpler. In 1974 Skordev delivered in Moscow several lectures about his generalization [25]. While working with it, Skordev noticed that the multi-valued functions satisfy several algebraic properties which are valid not only for the computability with multi-valued functions but also for many other types of computability (i.e., stochastic computability).

In autumn, the same year, Skordev defined the notion *compositional system*. In this definition he axiomatized the regularities he had noticed. This definition can be considered the direct precursor of the combinatory spaces he would define an year later. The definition of compositional system turned out to be too complex. Very few results were proven. The examples satisfying this definition, however, were amazingly many and varied – the paper [26] contained the impressive number of 13 examples satisfying the definition.

About the end of the same year Skordev was sent for a specialization in the USA – in the Stanford University for two months and in the University of California, Los Angeles for one month. During his stay he was able to find the correct formulation of the combinatory spaces [7, 9, 10]. The combinatory spaces were simpler, more powerful and more expressive than the compositional systems.

4.3. The language of the combinatory spaces

A well-known fact is that any function, definable by a λ-term, can be defined by an expression using only application and the two combinators S and K:

- $Sxyz = xz(yz)$,
- $Kxy = x$.

In essence, the translation of a λ-term into an expression, using only S and K, can be seen as a process of elimination of the bound variables. This is important, because in order to algebraize a certain formal language we need to eliminate the bound variables beforehand. For example, by eliminating the bound variables of the λ-calculus we obtain the language of the combinatory algebras (S and K). By eliminating the bound variables of the predicate calculus we obtain the cylindrical algebras.

Skordev had to do something similar. But while the λ-calculus and the combinatory logic are languages for untyped functions of higher order, Skordev needed an algebraization of a language with first-order functions only. While the language of combinatory algebras is based on the constants S and K and the operation application, the language of combinatory spaces, found by Skordev, includes the constants I, L, R, T, F and the operations composition, combination, branching, and iteration.

In order to facilitate the understanding we will describe the meaning of these constants and operations in the simplest case when the combinatory space consists of partial functions:

- I is the identity function: $I(x) = x$ for any x;
- T and F are constant functions, always returning values that can be interpreted as true and false;
- L and R are functions which, applied to a pair $\langle x, y\rangle$, return x and y, correspondingly;
- composition: $(fg)(x) = f(g(x))$;
- combination (pairing): $\langle f, g\rangle(x) = \langle f(x), g(x)\rangle$;
- branching: $(f \to g, h)(x) = \begin{cases} g(x), & \text{if } f(x) = \mathsf{true}, \\ h(x), & \text{if } f(x) \neq \mathsf{true}; \end{cases}$
- iteration of f controlled by g: $[f, g](x) = f^n(x)$, if $g(f^n(x)) \neq \mathsf{true}$ and $g(f^{n-1}(x)) = g(f^{n-2}(x)) = \cdots = g(f(x)) = g(x) = \mathsf{true}$.

Of course, anybody can declare that a particular set of operations are the "right" operations describing the abstract first-order computability, but without further substantiation such claims are void.

The reason the combinators S and K are special in the λ-calculus is the fact that they provide combinatory completeness. In order to demonstrate that the constants I, L, R, T, F and the operations composition, combination, branching, and iteration are special for the first-order abstract computability, we need to prove something analogous to the combinatory completeness of S and K.

Here the task is somewhat complicated, because there is not a single notion of abstract computability. In fact, there are three important notions of abstract computability. These are the prime and the search computability of Moschovakis [146] and the computability by means of recursively enumerable definitional schemes of Friedman [143] and Shepherdson [147].[4]

The fact that the prime computability can be characterized by computability in a combinatory space was already proven by Skordev in 1975 in the original paper about combinatory spaces [7]. For the search computability this was proven by Skordev in another paper of the same year [11]. The Friedman–Shepherdson computability turned out to be more difficult. The fact that it also can be characterized by computability in a combinatory space was proven by Ivan Soskov in 1987 [148].

4.4. The axioms

Just as any type of algebraic structures has a definition listing its axioms, so do the combinatory spaces.

Definition 4.1. A *combinatory space* is a partially ordered monoid, L, R, T and F are elements of the monoid, I is its identity, $\mathcal{C}$ is a subset of the monoid (intuitively, $\mathcal{C}$ contains the constant "functions"), a binary and a ternary operation combination $\langle\,.\,,\,.\,\rangle$ and branch $(\,.\,\to\,.\,,\,.\,)$ are defined and the following axioms are satisfied, where f, f', g, g', and h are arbitrary elements, and x and y are elements of $\mathcal{C}$:

- $\forall x(fx \geq gx) \implies f \geq g$;
- $\langle x, y\rangle \in \mathcal{C}$;
- $L\langle x, y\rangle = x$; $R\langle x, y\rangle = y$;
- $\langle f, g\rangle x = \langle fx, gx\rangle$;
- $\langle I, gx\rangle h = \langle h, gx\rangle$;
- $\langle x, I\rangle h = \langle x, h\rangle$;
- $T \neq F$;
- $Tx, Fx \in \mathcal{C}$;
- $(T \to g\,,\,g') = g$; $(F \to g\,,\,g') = g'$;

[4] In the special case of the ordinary computability over the natural numbers these three notions coincide.

- $h(f \to g\,,\, g') = (f \to hg\,,\, hg')$;
- $(f \to g\,,\, g')x = (fx \to gx\,,\, g'x)$;
- $(I \to gx\,,\, g'x)f = (f \to gx\,,\, g'x)$;
- if $f \geq f'$ and $g \geq g'$, then $(I \to f\,,\, g) \geq (I \to f', g')$.

Among all possible combinatory spaces, the so-called iterative ones turned out to be suitable for the development of the theory of computability.

Definition 4.2.

- Let f and g be some given elements of a combinatory space. An element i of it is said to be an iteration of f controlled by g, if the equality $i = (g \to if\,,\, I)$ holds and i belongs to each set which is closed under the mapping $\lambda h.(g \to hf\,,\, I)$ and can be represented as the intersection of sets of the form $\{h\colon d \geq ehx\}$, where $x \in \mathcal{C}$ and d, e are arbitrary elements of the combinatory space.[5]
- A combinatory space is *iterative*, if the iteration $[f, g]$ exists for any f and g.

This definition of the iteration enables reasoning about iteration in the spirit of D. Scott's μ-induction rule (cf., for example, de Bakker and Scott [140], de Bakker [141] or Hitchcock and Park [145]).

It is amazing how many important types of computability present examples of iterative combinatory spaces. For instance:

- The usual computability with partial functions over naturals.
- The computability with multi-valued functions over naturals.
- The Moschovakis' computability with partial or multi-valued functions in algebraic structure [146].
- Computability with infinite objects such as the real numbers or the elements of the Baire and Cantor spaces.
- Computability with error messages in case of unsuccessful computation.
- Various probabilistic models of computation.
- Computation with randomness.
- Computation with measurement of the complexity of the data processing.

[5] Besides this notion of iteration, Skordev used also a few weaker kinds of iteration. In the book [23] the iteration defined here is called "strong iteration".

- Computation with side effects. This means that the combinatory spaces capture not only the first-order functional style computability, but also the traditional procedural style computability. Notice that the axioms are not symmetric with respect to the left and the right argument of the combination $\langle f, g\rangle$. This is so in order to capture the possibility that the side effects during the computation of f can influence the subsequent computation of g.

A mathematical theory of a general nature must be justified by the existence of sufficiently many examples. But this is not enough. The theory must also contain serious mathematical results. This is the case with the theory of combinatory spaces. It contains a First Recursion Theorem:

Theorem 4.3 (First Recursion Theorem)**.** *In any iterative combinatory space, each element, recursively definable by composition, combination and branch is explicitly definable by composition, combination, branch and iteration.*

There are various normal form theorems:

Theorem 4.4. *In any iterative combinatory space, each element, recursively definable by composition, combination and branch, is explicitly definable by composition, combination and only one iteration.*

Important results (among others) are also Theorem for Existence of Universal Computable Elements, Second Recursion Theorem, Enumeration Theorem, abstract Rice and Rogers Theorems, a generalization of the Structured Program Theorem by Böhm and Jacopini [142]. Moreover, the corresponding results from ordinary recursive function theory and from Moschovakis' theory of abstract first order computability can be obtained as particular instances of general results.

Skordev has many publications about combinatory spaces [7–18, 20–22]. The interested reader, however, is advised to learn about the combinatory spaces from the two monographs of Skordev, the first one written in 1980 in Russian [19] and the second one written in 1992 in English [23].

5. Subrecursive analysis

After 2000 the main scientific interest of Skordev shifted towards subrecursive analysis. The motivation behind his first paper [32] in subrecursive analysis was provoked by his desire to find a definition for computable real number, which is as simple as possible and also suitable and comprehensible by undergraduate students, who presumably do not have practical experience in computability theory. Skordev managed to do that by using a representation of the computable real numbers by primitive recursive sequences of nested intervals with rational endpoints and length converging to zero. During the summer semester of 1999/2000 academic year, Skordev gave the course "Computable real numbers", which allowed bachelor's and master's students to get acquainted with the state of the art in this topic. It inspired interest even in students, who were more inclined towards practical applications. In

2001, Branimir Lambov defended successfully his master's thesis under Skordev's supervision, in which he created his own system for exact real arithmetic by using the theoretical framework of the so-called partial approximations, introduced by Skordev in [32].

In his subsequent studies, Skordev focused on applications of increasingly smaller subrecursive classes to computable analysis. At this moment, the restriction of the generality of computations in this field had traditionally been oriented towards complexity classes from discrete complexity theory, which are defined using limitations on the resources exploited in the computations. Applications of subrecursive classes with inductive definitions in the spirit of Grzegorczyk's hierarchy were not well developed. In his paper [33], Skordev set himself the goal to describe sufficient conditions for a class of functions $\mathcal{F}$, so that the set of $\mathcal{F}$-computable real numbers is a field (closed under arithmetic operations). Skordev also managed to show that if the class $\mathcal{F}$ is additionally closed under bounded minimization, this field is real-closed, that is, it contains the real roots of the polynomials with $\mathcal{F}$-computable coefficients (these conditions on the class $\mathcal{F}$ are satisfied by all Grzegorczyk classes $\mathcal{E}^m$ for $m \geq 2$). This can be regarded as a relativization of a known result of Rice that the computable complex numbers form an algebraically closed field. The interesting question arises whether these results hold uniformly, that is, given representations of the coefficients of the polynomial, to construct effectively a representation of the root. Skordev assigned this topic to his master's student Peter Peshev, who defended his thesis in 2005. It turned out that the effective transformation described above is possible using the class of $\mathcal{E}^2$-computable operators, on the stipulation that the root produced by the operators depends not only on the coefficients, but also on their representations. In other words, applying the operators to different representations of the same coefficients may produce different roots. The low complexity of the obtained operators motivated Peshev and Skordev to publish their results in the paper [52], presented at the Computability in Europe conference in Swansea (England) in 2006.

The above considerations clearly imply that the set of $\mathcal{E}^2$-computable real numbers contains all algebraic real numbers. In order to show that there exist transcendental $\mathcal{E}^2$-computable real numbers, Skordev began to study the subrecursive complexity of concrete famous constants. His first results in this direction are contained in the paper [34], where he proved the $\mathcal{E}^2$-computability of e, π, Liouville's number L and Euler-Mascheroni's constant. Shortly after that, in [35], he simplified substantially his proofs and managed to formulate a general method, which was relatively easy to apply for showing the $\mathcal{E}^2$-computability of many other constants. In addition, it became clear that his method used only bounded summation (and not bounded primitive recursion), which made it suitable for the class $\mathcal{L}^2$ of Skolem's lower elementary functions.

In 2008, Andreas Weiermann from Ghent University in Belgium noticed that Skordev's results on the constants e and L can be strengthened to $\mathcal{M}^2$-computability, where $\mathcal{M}^2$ is the class of all polynomially bounded functions definable in bounded arithmetic. In particular, Weiermann used the definability of the graph of the factorial function in bounded arithmetic. It is interesting to note that Skordev had already studied the same class $\mathcal{M}^2$ with another equivalent definition in [50].

Skordev assigned to his last PhD student Ivan Georgiev the task to find new constants, for which the described methods for proving $\mathcal{L}^2$- and $\mathcal{M}^2$-computability are applicable. Georgiev managed to devise another method for proving $\mathcal{M}^2$-computability of real numbers, based on the non-trivial fact that $\mathcal{M}^2$ is closed under log-bounded summation. As a corollary, using a proper fast converging representation, the number π was shown to be $\mathcal{M}^2$-computable. Interestingly, the question for the $\mathcal{M}^2$-computability of Euler-Mascheroni's constant remained open until 2017, when Georgiev found a suitable sub-exponential method for integration of analytic functions.

The new results on $\mathcal{M}^2$-computability were presented in the paper [39] by Skordev, Weiermann and Georgiev. Its main goal was to show that the field of $\mathcal{M}^2$-computable real numbers is closed under the elementary functions of calculus. In this way, the focus of research gradually shifted towards subrecursive computability of real functions. It turned out that the elementary functions of calculus are uniformly computable with respect to the class of $\mathcal{M}^2$-substitutional operators, but after restricting them to compact domains. In order to obtain computability of the elementary functions of calculus on their whole domains, Skordev and Georgiev study in [37,41] the broader notion of conditional computability of real functions, in which the computing procedure depends not only on the representations of the real arguments, but also on an additional natural parameter, whose value is found by possibly unlimited minimization, controlled by a property of appropriate kind. All elementary functions of calculus are conditionally computable with respect to the class of $\mathcal{M}^2$-substitutional operators.

Closely related and similar in motivation is the work of Katrin Tent from the University of Münster, Germany and Martin Ziegler from the University of Freiburg, Germany. They also consider two notions for relative computability of real functions (with respect to the class $\mathcal{L}^2$), but their approach is different. They use in a more direct way the rational approximations of the arguments of the real functions, and thus they circumvent the use of total functions in the natural numbers as names. Skordev managed to prove in [42] that our uniform computability is equivalent to the uniform variant of the computability of Tent and Ziegler (in this way he strengthened some of their results from $\mathcal{L}^2$ to $\mathcal{M}^2$). A substantial part of Georgiev's PhD thesis, successfully defended in 2016 under Skordev's supervision, was a similar characterization theorem for the conditionally computable real functions.

The notion of uniform computability of real functions exploits total operators, therefore it is stronger than the generally accepted notion for a computable real function, where the operators may not be defined on total functions, which are not names of real numbers in the domain of the real function. Skordev found in [44] a similar characterization for this general computability, avoiding infinitary names, by application of his previously introduced approximation nets. These nets define a natural, but logically complex, correspondence between the proximity of rational numbers to the arguments and to the value of the real function. It turns out that a real function is computable if and only if it has a recursively enumerable approximation net. In [43] Skordev substantially generalizes this result for functions in arbitrary topological spaces with a fixed countable base and arbitrary metric spaces with a fixed everywhere dense countable subset.

One of the last topics that Skordev developed in [46–48], is an elegant application of his theory of combinatory spaces to questions on computability in represented and multi-represented spaces in the sense of computable analysis. He proved that the computability in these spaces, that is, the existence of computable realizers, is preserved by relative computability in certain iterative combinatory spaces, connected to the absolute prime computability, introduced by Moschovakis in [146]. This result combined with Theorem 4.3 gives a powerful new method for proving computability of real functions.

6. Boolean functions and other works

Skordev has an interesting result about the functionally complete sets of Boolean functions.

Definition 6.1. An n-argument Boolean function f is a *subfunction* of an m-argument Boolean function g, if for some natural numbers $k_1, k_2, \ldots, k_m$, such that $\{k_1, k_2, \ldots, k_m\} = \{1, 2, \ldots, n\}$, the following identity holds:

$$f(x_1, x_2, \ldots, x_n) = g(x_{k_1}, x_{k_2}, \ldots, x_{k_m}).$$

If in addition $m > n$, then f is a *proper subfunction* of g.

For example $f(x, y) = \overline{x \vee y}$ is a proper subfunction of $g(x, y, z) = \overline{x \vee y \vee z}$ because $f(x, y) = g(x, y, y)$.

Definition 6.2.

1. A *functionally complete set* of Boolean functions is one which can be used to express all possible Boolean functions by combining members of the set into an expression.

2. A *basis* is a minimal functionally complete set, i.e., a functionally complete set, whose proper subsets are no longer functionally complete.

3. A basis is *primitive* if none of the functions in it can be replaced by proper subfunctions without spoiling the completeness of the set.

For example the one-element sets $\{f\}$ and $\{g\}$, where f and g are defined as above, are bases. The set $\{f\}$ is a primitive basis while $\{g\}$ is not.

Theorem 6.3 (2008 [54, 55]). *There are* 44 *primitive bases, such that any functionally complete set of Boolean function can be reduced to a primitive basis of those* 44*, by omitting some functions or replacing them by their subfunctions.*

Among many others, Dimiter Skordev has papers on loop detection in computer programs [59, 60, 62–65], a paper where he constructs a λ-algebra [74], an algebraization of the flow diagrams [66] and papers where he shows how multi-valued homomorphisms can be used for program verification [56, 58].

The Historical Notes [117] of Skordev are a unique source of information on the development of mathematical logic in Bulgaria.

7. Epilogue

The immortal spirit of a man desires to do much, yet alas, the life of a man is brief and insufficient. The greatness of a human life is measured not by its length but by the legacy left behind. Dimiter Skordev bequeathed to us not only his scientific works but also the knowledge and wisdom he imparted to his students who continued his work. The world began talking about the "Bulgarian school" in computability theory.

Dimiter Skordev was embodiment of precision, delicacy and ethics. Over the years, we have been molded by his exemplary qualities. We can only hope that we have taken a small spark out of his excellent fire, a spark which has made us work to some extent along the lines that he desired.

We will conclude with the beautiful words that Skordev once said at the funeral of his teacher, Prof. Tagamlitzki.[6] Words that we now repeat and apply to our own teacher:

> He has left us two examples – a model of a scientist and a model of a person. Emulating his first example is a privilege granted to only a select few. However, his second example – serving science and society with honesty, selflessness and dedication – is a goal attainable by each of us.

May his examples guide us to uphold the highest standards of science and ethics in our own journeys!

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